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→ Edith see next page

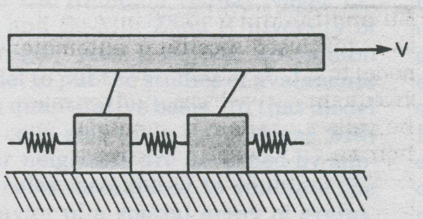
ARE FRACTURES FRACTAL OR QUAKES CHAOTIC?

Just over one year ago, as San Franciscans were settling down to an evening of home runs and strikeouts, they got instead a few moments of shock and terror. The earthquake that struck them was not totally unexpected for that general area, but it certainly had not been pinpointed to the exact day and location. Earthquakes still defy accurate prediction, despite intensive efforts to understand their complex underlying dynamics.

Now some analysts are wondering whether earthquakes can best be understood as fractal or chaotic phenomena. They are finding that some very simple and sometimes symmetric models can exhibit surprisingly complex, asymmetric and chaotic behavior. If the models prove to be appropriate representations of the real movements of the Earth's crust over fault zones, they may help determine the important parameters or lend insight into the patterns of recurrence. On the down side, if earthquakes, like some of the models, are found to be deterministically chaotic, the exponential divergence of solutions would preclude long-range forecasts, although the equations might facilitate predictions in the short term.

Fractal structures

Even before Benoit Mandelbrot coined the term "fractal" in 1982 to describe the similarity of spatial patterns of some phenomena on many different scales, geologists recognized that earthquake zones have some characteristics now associated with fractals. Beno Gutenberg and Charles Richter at Caltech observed in the early 1940s that the frequency of occurrence $f(m)$ of earthquakes greater than a given magnitude m is given by $\log_{10} f(m) = c - bm$, where b and c are constants. The magnitude is in turn related to the log of the moment, so this relation is essentially a power law. The law holds for magnitudes ranging over several powers of 10, and the slope b is near 1 for fault zones worldwide. Such a relationship indicates that small events follow the same laws as large events, and thus implies that all events must be governed by a scale-invariant mechanism. That is the essence of fractal behavior. The magnitude of an earthquakes is related to its area,[1] so that the self-similarity implied by the Gutenberg–Richter law is related to spatial dimensions as well.



Blocks and springs dragged along at a constant velocity form a class of models often used to simulate the sticking and slipping motion of Earth's motion along a fault line.

Thus analysts have known that fault zones obey fractal statistics, but they haven't known why. Last year, with a simple block-and-spring model, Jean Carlson (University of California at Santa Barbara) and James Langer (Institute of Theoretical Physics at UCSB) reproduced behavior akin to the Gutenberg–Richter relation between magnitude and frequency.[2] Their model is based on one first devised in 1967 by Robert Burridge (now at Schlumberger–Doll Research, London) and Leon Knopoff (UCLA) to represent the sticking and slipping of the Earth's crust as it moves along a fault zone. The blocks are connected to one another by springs, and they are also connected, by other springs, to a constant-velocity drive that pulls the chain of blocks. (See the figure above.) As the drive pulls on the blocks, they stick until the spring force exceeds the static friction force, and then one or more blocks slides all at once until the spring force is eased. Burridge and Knopoff studied both physical and computer models of these chains of blocks. Computer capacity limited them to 10 blocks at that time. John Rundle (Lawrence Livermore National Laboratory) told us that these and other early investigators already saw some evidence for power-law behavior.

Carlson and Langer extended the chain of blocks to include as many as 200 blocks, with the chain represented by an array of coupled differential equations. In Carlson and Langer's model the friction force decreases with increasing block speed. The nonlinearity stems from the abrupt transition between static and dynamic friction. Once the system reached a statistically stable state, Carlson and Langer recorded the velocity of each block as a function of time. Interspersed between events in which only a single block slid at low speed were larger events involving many blocks reaching speeds that exceeded the velocity at which the chain was being dragged. Occasionally there was a "catastrophic" event, with all the blocks sliding at once. The Santa Barbara theorists note that small events, in which one or a few blocks slide between others that remain fixed, tend to equalize the distance between the blocks and hence "smooth" the system. As a consequence, a larger group of blocks is likely to slip together the next time. Thus the smoothing paves the way for larger events. Also, because the friction is smaller for faster-moving blocks, any inhomogeneities in a group of slipping blocks get amplified as they slide, and smaller events are constantly generated.

To summarize the statistics of the events of different sizes, Carlson and Langer defined a "moment" for each event that was a measure both of the number of blocks involved and of their displacements, and a "magnitude" that was the log of the moment. A histogram of the magnitude m on a log plot showed that the number $N(m)$ of events followed a power law of the Gutenberg–Richter type with the slope about equal to 1. (See the figure below.) For events of large magnitude, a striking departure from the Gutenberg–Richter law occurred: The block system experienced more large slides than the simple formula would indicate.

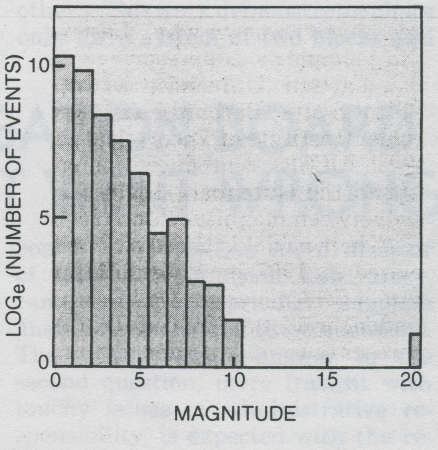
Carlson told us that simple models such as theirs can help identify the important physical parameters for earthquake occurrence and provide guidelines for estimating the regularity of the repeat times. However, she points out that the model is a simplification of the faulting process, and the connection to individual earthquakes is still tenuous.

Self-organized criticality

Carlson and Langer note that their model of earthquakes is an example of "self-organized criticality," a concept originated[3] by Per Bak (Brookhaven National Laboratory), Chao Tang (now at UCSB) and Kurt Wiesenfeld (now at Georgia Tech) in 1987. Indeed, Bak and his colleagues have often cited earthquakes as an example of self-organized criticality. According to their concept, certain systems are marginally stable and, when perturbed from equilibrium, will evolve naturally back toward the marginally stable state. In other words, such a system organizes itself to be perpetually in a critical state. The three theorists proposed that this mechanism of self-organized criticality might underlie all systems that exhibit "$1/f$" noise, that is, systems whose power spectra vary as $f^{-\beta}$, with β near 1, over a large range of frequencies. The power law identified by Gutenberg and Richter puts earthquakes in this category of phenomena.

As an example of self-organized criticality, consider a pile of sand. The system cannot be stable against small perturbations if the pile is so steep that one additional grain of sand will collapse the entire pile. Sand piles of low slope are more stable: Extra grains of sand may induce only small slides. The sand pile then consists of regions within which the perturbation can propagate, but these regions do not communicate with one another, so the slides do not extend to infinte distances. If a sand pile is built by slowly adding sand, it will neither go to the steep, unstable state nor to the shallow state with small avalanches, but will eventually evolve to a critical state separating those extremes, in which avalanches of all sizes can occur. At the critical state, the pile remains at the same height on average, because the addition of new grains of sand is balanced by occasional avalanches of various sizes. (This state is called "critical" to suggest the analogy with critical phenomena, which exhibit correlations of all sizes.) For this system, Bak, Tang and Wiesenfeld assert, there is no longer a natural length scale. The variation in the sizes of the avalanches corresponds to the fluctuation in the power output of the system.

Bak, Tang and Wiesenfeld, and later Leo Kadanoff, Sidney Nagel, Lei Wu and Su-Min Zhou (University of Chicago),[4] used a cellular automaton model to put the studies of avalanches on a quantitative basis. In that model the cells and their interactions with their neighbors are governed by simple rules formulated to simulate the behavior of a spatial array of grains. The state of each cell is given by an integer value that represents the local slope of the pile at that point. To add a grain to the pile one simply adds an integer to a randomly selected cell. If the sum exceeds some critical value, it is decreased by a certain amount, say 4, while the count in each adjacent cell is increased by 1. Thus, if many cells in a given area are near the critical value, the perturbation can trigger an avalanche involving many grains. Bak and his colleagues started each run of their model with the "slopes" distributed randomly. They then periodically added grains to random cells. Once the system had been driven for a while in this way, they would examine the size of the avalanche that the next grain would instigate—that is, the number of cells affected by the perturbation. The team found a power-law distribution of events as a function of cluster size and time scale.



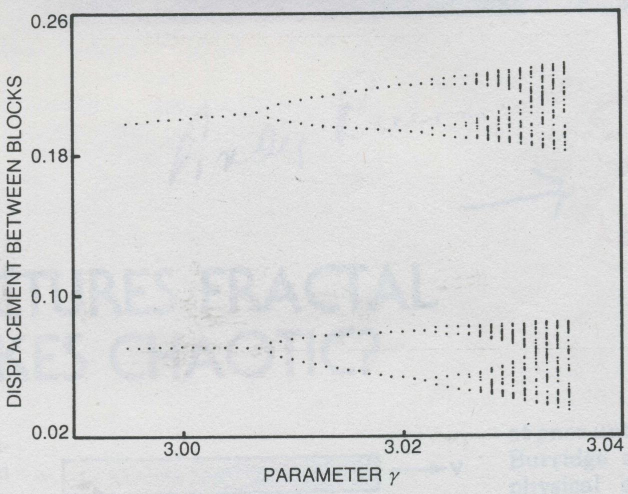
Distribution of magnitudes of earthquakes simulated by a block-and-spring model.[2] This distribution resembles that given by the Gutenberg–Richter law for real earthquakes, but the simulation predicts a larger number of higher-magnitude events.

The cellular automaton models of avalanches can just as easily simulate the behavior of an array of blocks connected by springs, if the integer for each cell denotes the strength of the spring force and the threshold value is the maximum static friction force. Bak and Tang, in fact, used such a model with up to 8 000 blocks in two- and three-dimensional arrays to study the self-organized criticality of earthquakes,[5] and found that the distribution of events with a certain energy release E varied as $E^{-\tau}$. They see no deviation at large magnitude. Bak feels that the scaling behavior explains why one cannot predict large earthquakes very well, but notes more hopefully that it also indicates that one can infer information about large earthquakes (for which real data are scarce) by studying the patterns of small ones. Bak feels that earthquakes or other systems in the critical state are just on the edge of chaos, so that long range forecasts are not precluded as much as they would be if the system were chaotic.

A similar model of earthquakes has been created by Stephen Brown (Sandia Laboratories), Rundle and Christopher Scholz (Columbia University's Lamont–Doherty Geological Observatory). While Bak and Tang look only at samples of their model far from the boundaries, Brown, Rundle and Scholz examine the effects of finite boundaries. Like Carlson and Langer, they find that larger events occur more frequently than one would infer by extrapolation from smaller events. Scholz notes that earthquakes occur in relatively shallow regions of the earth's crust. Seismologists have long been aware of differences between earthquakes small enough not to reach the edges of the bounding region and those that are large enough to be constrained by this dimension. Thus there is a characteristic length that breaks the scaling between these two types of earthquakes.

Chaos

While the models mentioned so far involve large arrays of blocks, others explore the dynamics of just a few blocks. Andy Ruina (Cornell University) and other researchers have simulated earthquakes with single block-and-spring models subject more to realistic friction forces that depend both on the speed of the block and on its past history of movement (so-called



Period doubling in the behavior of two elastically coupled blocks with equal masses but subject to different friction forces.[9] The graph plots the displacement between the two blocks at the end of a slip as a function of a system parameter γ. As γ increases, the system tends toward two phase-space points, then four and so forth.

rate- and state-variable friction). The parametrization of the friction force in Ruina's model was based largely on the experimental measurements of James Dieterich (US Geological Survey, Menlo Park, California). With Jeffrey Nussbaum (now at General Electric) Ruina decided in 1987 to explore the consequences of the simplest possible model to see whether the dynamics alone rather than fault heterogeneities could produce spatially asymmetric behavior.[6] The answer was yes. Nussbaum and Ruina's model consisted of two identical blocks, both subject to a friction force that switched between two possible values, dynamic and static, as the blocks first slid and then stuck. They found that the motion of the blocks was periodic, but generally not symmetric: In some solutions one block would always move in quicker slips than the other, while in other modes the two blocks would alternately undergo short slips.

In a more recent paper, Ruina and Frank Horowitz (now at Northwestern University) analyze a model with rate- and state-variable friction that is homogeneous in its material properties and has no effects from the ends of a chain.[7] It is essentially a continuum model with the slip varying continuously along the fault. Again they find that the dynamics can generate both temporal and spatial complexity in seismic phenomena.

This year Jie Huang and Donald Turcotte adopted[8] essentially the same model as Nussbaum and Ruina, but they allowed the friction force acting on one block to be larger than that on the second by a factor β. With this asymmetry, Huang and Turcotte produce evidence for deterministically chaotic behavior. For their analysis they first displayed the successive states of their system in phase space, plotting the position of one block versus the other. The evolving trajectory never settled down to one or a finite number of points, as one would expect if the system approached either steady-state or periodic behavior. Rather, it filled the phase-space plot. Moreover, the system appeared to follow a period-doubling route to chaos: As a parameter γ, which describes the variation of friction with velocity, was increased, the system evolved first toward a single phase-space point; then, as the parameter continued to increase, the system oscillated between two final states, then four states, then eight and so forth.[9] (See the figure above.)

Turcotte told us that they have now calculated a positive value for the Lyapunov exponent for a model in which the friction becomes smaller as the block moves faster. (The Lyapunov exponent is a measure of the rate of exponential divergence of two points of the system in phase space that start out arbitrarily close to each other.) This work demonstrates chaos only for a system of two blocks and not for the far more complex structure of faults and cracks within the Earth's crust. It remains to be seen whether this chaotic behavior in a low-order model implies chaotic behavior in real systems of higher order.

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References

1. K. Aki, in *Earthquake Prediction*, D. W. Simpson, P. G. Richards, eds., Am. Geophys. Union, Washington, D.C. (1981), p. 566.
2. J. M. Carlson, J. S. Langer, Phys. Rev. Lett. **62**, 2632 (1989).
3. P. Bak, C. Tang, K. Wiesenfeld, Phys. Rev. Lett. **59**, 381 (1987); Phys. Rev. A **38**, 36 (1988).
4. L. Kadanoff, S. Nagel, L. Wu, S. Zhou, Phys. Rev. A **39**, 6524 (1989).
5. P. Bak, C. Tang, J. Geophys. Res. **94**, 15 635 (1989).
6. J. Nussbaum, A. Ruina, Pure Appl. Geophys. **125**, 629 (1987).
7. F. G. Horowitz, A. Ruina, J. Geophys. Res. **94**, 10279 (1989).
8. J. Huang, D. L. Turcotte, Geophys. Res. Lett. **17**, 223 (1990).
9. J. Huang, D. L. Turcotte, to be published in Pure Appl. Geophys. (1990).

HUBBLE INVESTIGATION BOARD FINDS OUT WHAT WENT WRONG

By what sequence of mishaps did the Hubble Space Telescope acquire its unfortunate spherical aberration? And how did this severe optical flaw escape notice until after the HST was launched into orbit last spring? These were the two principal questions set out for the HST Optical Systems Board of Investigation convened by NASA at the end of June, shortly after it became clear that the Hubble telescope was hobbled by half a wavelength of spherical aberration. (See PHYSICS TODAY, August, page 17.)

In less than three months the board, headed by Lew Allen, director of the Jet Propulsion Laboratory, came up with a surprisingly complete answer to the first of these questions. The Allen board's answer to the second question, more fraught with touchy issues of administrative responsibility, is expected with the release of its full report sometime this month.

The other members of the board are Roger Angel and Robert Shannon (both at the University of Arizona), Charles Spoelhof (retired from Kodak), George Rodney (NASA headquarters) and John Mangus (Goddard Space Flight Center).

On 13 September, after the board's third meeting at Hughes Danbury Optical Systems (formerly Perkin-Elmer) in Danbury, Connecticut, where the Hubble's great primary mirror had been painstakingly polished to its final figure a decade ago, Allen released a statement outlining the circumstances that resulted in the crucial fault in the optical template that guided, or rather misguided, the careful polishing of the mirror. "The board is confident," he wrote, "that